



Keldysh Institute of Applied Mathematics (Russian Academy of Sciences)

The Fuzzy Origin–Destination Matrix Estimation for Planning Air Traffic

Vladimir Sudakov

sudakov@ws-dss.com

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Problem definition



Aviation has quite detailed statistics on air transportation between individual airports.

In Russia (in the form of 14GA), in the U.S. (in the T-100 data bank) these data are collected monthly by aviation authorities.

But it is problematic to understand how many passengers are forced to transit.

Fuzzy Origin–Destination Matrix

The flow along the arc d is equal to the sum of all correspondences from i to j through d :

$$V_d = \sum_i \sum_j p_{ij}^d x_{ij} \quad \forall d \in D$$

$$X_{ij} = \langle x_{ij}^{\min}, \bar{x}_{ij}, x_{ij}^{\max} \rangle$$

$$x_{ij}^{\min} = \min_{x_{ij}} x_{ij}$$

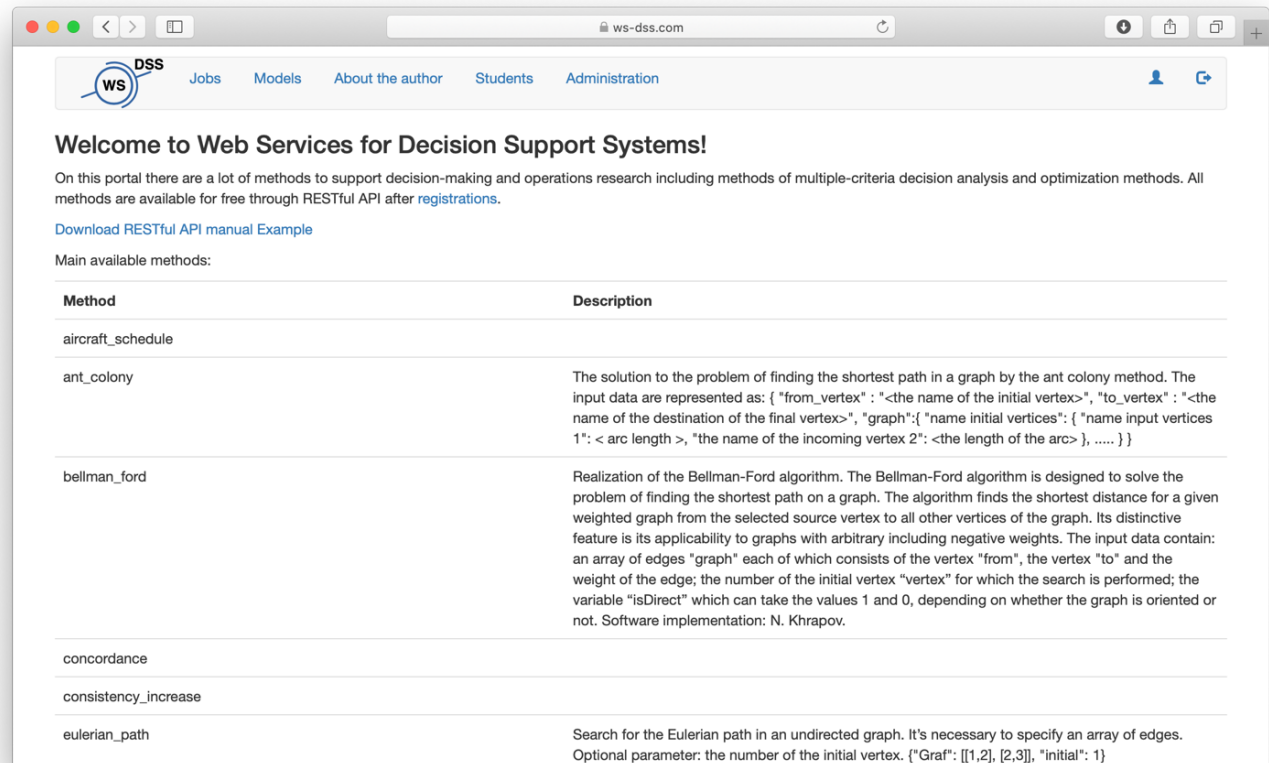
$$x_{ij}^{\max} = \max_{x_{ij}} x_{ij}$$

$$\bar{x}_{ij} \cong \alpha_{ij} x_{ij}^{\max} + (1 - \alpha_{ij}) x_{ij}^{\min}$$

$$\min_{\bar{x}_{ij}} \max_{i,j} |\bar{x}_{ij} - \tilde{x}_{ij}|$$

WS-DSS.COM

- Ruby on Rails
- PostgreSQL
- Implementation of models in Python, Ruby, R, C ++
- SCIP, metis, nlopt optimizers
- RESTful API integration with JSON and console methods via



General algorithm

The web server WS-DSS receives an HTTP POST request with the source data for the task being solved.
In response, a unique ID of the created task is returned to the client.

WS-DSS forms the task for calculation in Sidekiq.

The Sidekiq workflow starts the calculation module in Python

An array of traffic volumes V_d through all arcs d is formed.

With the help of the NetworkX package using Dijkstra's algorithm all shortest paths between all nodes are calculated.

Filling in p_{ij}^d parameters. For all d it is checked if it is included in the shortest path from node i to node j . If the answer is yes, then $p_{ij}^d = 1/k$, where k is the total number of shortest paths from i to j .

Calculation x_{ij}^{\min} . Formation of the target min function solution of the obtained optimization task for all combinations of i and j values.

Calculation of x_{ij}^{\max} . Formation of the target max function solution of the obtained optimization task for all combinations of i and j values.

Search \bar{x}_{ij} by solving optimization.

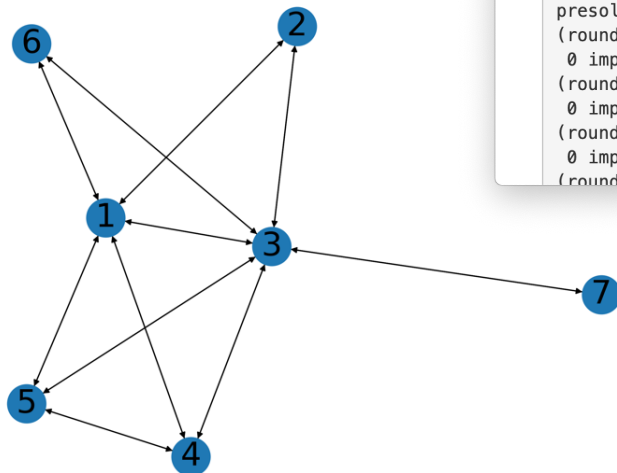
Save the obtained $\langle x_{ij}^{\min}, \bar{x}_{ij}, x_{ij}^{\max} \rangle$ in a PostgreSQL database.

Returning the received solution by HTTP request of GET client with ID task. If the solution is not yet received at the time of the request, a special status of waiting for the result will be returned.

MODEL EXAMPLE

The testing of the system was carried out according to the data on air traffic in Russia between 7 airports:

1. Ekaterinburg,
2. Mineralnye Vody,
3. Moscow,
4. Omsk,
5. Samara,
6. Syktyvkar,
7. Makhachkala.



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User: sudakov@ws-dss.com
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Method: fuzzy_od_matrix
User: sudakov@ws-dss.com
Input data:

```

1,2,3933
1,3,96707
1,4,293
1,5,1693
1,6,439
7,3,51188
2,1,4073
2,3,71620
3,1,93759
3,7,53810
3,2,76650
3,4,40848
3,5,61964
3,6,12123
4,1,359
4,3,41144
4,5,212
5,1,2105
5,3,62557
5,4,149
6,1,444
6,3,13104

```

Output data:

```

presolving:
(round 1, fast)
0 impls, 0 clqs
(round 2, fast)
0 impls, 0 clqs
(round 3, fast)
0 impls, 0 clqs
(round 4, fast)

```

ws

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```

SCIP Status      : problem is solved [optimal solution found]
Solving Time (sec) : 0.01
Solving Nodes     : 1
Primal Bound      : +2.83239000000000e+05 (1 solutions)
Dual Bound       : +2.83239000000000e+05
Gap               : 0.00 %

```

| i | j | x* | yp | yn | xmin | x_arg | xmax |
|---|---|-------|-------|-------|-------|-------|-------|
| 1 | 2 | 2479 | 0 | 0 | 1025 | 2479 | 3933 |
| 4 | 2 | 179 | 0 | 0 | 0 | 179 | 359 |
| 5 | 2 | 1052 | 0 | 0 | 0 | 1052 | 2105 |
| 6 | 2 | 222 | 0 | 0 | 0 | 222 | 444 |
| 1 | 3 | 83211 | 0 | 13409 | 42897 | 69802 | 96707 |
| 1 | 7 | 13495 | 13409 | 0 | 0 | 26905 | 53810 |
| 1 | 4 | 97 | 48 | 0 | 0 | 146 | 293 |
| 2 | 4 | 146 | 0 | 0 | 0 | 146 | 293 |
| 6 | 4 | 48 | 97 | 0 | 0 | 146 | 293 |
| 1 | 5 | 772 | 74 | 0 | 0 | 846 | 1693 |
| 2 | 5 | 846 | 0 | 0 | 0 | 846 | 1693 |
| 6 | 5 | 74 | 148 | 0 | 0 | 222 | 444 |
| 1 | 6 | 86 | 133 | 0 | 0 | 219 | 439 |
| 2 | 6 | 219 | 0 | 0 | 0 | 219 | 439 |
| 4 | 6 | 59 | 119 | 0 | 0 | 179 | 359 |
| 5 | 6 | 73 | 146 | 0 | 0 | 219 | 439 |
| 7 | 3 | 8531 | 17062 | 0 | 0 | 25594 | 51188 |
| 7 | 1 | 12724 | 12869 | 0 | 0 | 25594 | 51188 |
| 7 | 2 | 8531 | 17062 | 0 | 0 | 25594 | 51188 |
| 7 | 4 | 6808 | 13616 | 0 | 0 | 20424 | 40848 |
| 7 | 5 | 8531 | 17062 | 0 | 0 | 25594 | 51188 |
| 7 | 6 | 6061 | 0 | 0 | 0 | 6061 | 12123 |
| 2 | 1 | 2860 | 0 | 0 | 1648 | 2860 | 4073 |
| 2 | 3 | 62651 | 0 | 17936 | 17810 | 44715 | 71620 |
| 2 | 7 | 8968 | 17936 | 0 | 0 | 26905 | 53810 |
| 3 | 1 | 81034 | 0 | 12869 | 42571 | 68165 | 93759 |
| 3 | 7 | 8968 | 17936 | 0 | 0 | 26905 | 53810 |
| 4 | 7 | 6857 | 13714 | 0 | 0 | 20572 | 41144 |
| 5 | 7 | 8968 | 17936 | 0 | 0 | 26905 | 53810 |
| 6 | 7 | 6552 | 0 | 0 | 0 | 6552 | 13104 |
| 3 | 2 | 68118 | 0 | 17062 | 25462 | 51056 | 76650 |

The fuzzy origin–destination matrix allows to solve the following problems:

- Optimal route planning
- Rational scheduling
- Assessment of aircraft import substitution prospects
- Analysis of possibilities to replace outdated aircraft equipment
- Support for decision making on modernization of existing transport infrastructure

Prospects

Let's integrate your models

Write me: sudakov@ws-dss.com

Thanks for attention!